

# The puzzle of the non-planar structure of the QCD contributions to the Gottfried sum rule<sup>1</sup>

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## ABSTRACT

The recent finding, that in QCD at the  $O(\alpha_s^2)$ -level only non-planar diagrams, which are suppressed by a factor  $1/N_c^2$  relative to planar ones, are contributing to the valence part of the Gottfried sum rule, is described. To our knowledge, this intriguing unique feature did not manifest itself previously in any perturbative expansion of any gauge quantum field model. We hope that this discovery may have some theoretical explanation.

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## 1. Introduction.

The studies of the lepton-nucleon deep-inelastic (DIS) sum rules provide important information about the structure of QCD both in perturbative and non-perturbative sectors. Moreover, their more detailed investigations are continuing to reveal new intriguing questions for further theoretical and phenomenological explanations. Among them are the results of the recent work of Ref. [1], which is devoted to the comparison of the obtained therein QCD predictions for the Gottfried sum rule [2] with the the expression for the Adler sum rule [3] within the large- $N_c$  expansion.

## 2. Gottfried and Adler sum rules: the definitions.

Consider first the isospin Adler sum rule, expressed through the structure function  $F_2^{\nu N}$  of neutrino-nucleon DIS as

$$I_A = \int_0^1 \frac{dx}{x} \left[ F_2^{\nu p}(x, Q^2) - F_2^{\nu n}(x, Q^2) \right] = 4I_3 = 2 \quad . \quad (1)$$

In terms of parton distributions Eq. (1) takes the following form

$$I_A = 2 \int_0^1 dx [u_v(x) - d_v(x)] = 2 \quad (2)$$

where  $u_v(x) = u(x) - \bar{u}(x)$  and  $d_v(x) = d(x) - \bar{d}(x)$  are the valence parton distributions of light quarks. It is possible to show, that  $I_A$  receives neither perturbative nor non-perturbative ( $1/Q^2$ )-corrections [4]. In view of this the Adler sum rule is  $Q^2$  independent and demonstrates the consequence of the property of scaling [5]. This property (or so called “automodelling” behaviour of structure functions) was rigorously proved in the case of charged lepton-nucleon DIS by N.N. Bogolyubov and coauthors [6] with application of general principles of local quantum field theory, described e.g. in the classical text-book [7]. However, it is known, that in QCD scaling is violated. The sources of its violation are related to the asymptotic freedom effects, discovered within renormalisation-group concept [8] in the papers of the 2004 Nobel Prize laureates [9], and to non-perturbative contributions. Both types of these effects manifest themselves (though is some puzzling way) in the analog of the Adler sum rule, namely in the Gottfried sum rule. It can be defined as the first  $N = 1$  non-singlet (NS) Mellin moment of the difference of  $F_2$  SFs of DIS of charged leptons on proton and neutron, namely

$$I_G^v = \int_0^1 \frac{dx}{x} \left[ F_2^{lp}(x, Q^2) - F_2^{ln}(x, Q^2) \right] = \frac{1}{3} \int_0^1 dx \left( u_v(x, Q^2) - d_v(x, Q^2) \right) \quad . \quad (3)$$

The definition of Eq. (3) is presented in the case of assumption accepted previously that the sea quarks are flavour-independent. It corresponds to the condition  $\bar{u}(x, Q^2) = \bar{d}(x, Q^2)$ , accepted in the early works on the subjects. However, due to the appearance of experimental data for the muon-nucleon DIS, Drell-Yan process and semi-inclusive DIS we know at present that this condition is violated and  $\bar{u}(x, Q^2) < \bar{d}(x, Q^2)$  ( for reviews see, e.g. [10]-[12]). Therefore, the definition of the Gottfried sum rule should be modified as:

$$I_G = \int_0^1 \frac{dx}{x} \left[ F_2^{lp}(x, Q^2) - F_2^{ln}(x, Q^2) \right] = I_G^v + \frac{2}{3} \int_0^1 dx \left( \bar{u}(x, Q^2) - \bar{d}(x, Q^2) \right) \quad , \quad (4)$$

where the last term has non-perturbative origin and is related to the manifestation of isospin-breaking effects in the Dirac sea. We will return to its discussion later on, after describing main puzzle, discovered in Ref. [1], that in the  $O(\alpha_s^2)$  level the scaling violation corrections to Eq. (3) have typically non-planar structure, namely in the large- $N_c$  limit [13] (where  $N_c$  is the number of colours) they are suppressed by a  $(1/N_c^2)$  factor. This means, that the leading in  $N_c$  planar diagrams are cancelling out in the analysed QCD corrections to the valence contribution  $I_G^v$ . We hope that this discovery may have some theoretical and phenomenological explanations.

### 3. Large $N_c$ -expansion and the relation between Gottfried and Adler sum rules.

Let us now support the statements made in the previous Section by more formal considerations, presented in Ref. [1]. The solution of the renormalization group equation for the valence contribution  $I_G^v$  to the Gottfried sum rule has the following form

$$I_G^v = A(\alpha_s)C^{(l)}(\alpha_s) \quad (5)$$

with the anomalous-dimension term

$$A(\alpha_s) = 1 + \frac{1}{8} \frac{\gamma_1^{(N=1)}}{\beta_0} \left( \frac{\alpha_s}{\pi} \right) + \frac{1}{64} \left( \frac{1}{2} \frac{(\gamma_1^{(N=1)})^2}{\beta_0} - \frac{\gamma_1^{(N=1)} \beta_1}{\beta_0^2} + \frac{\gamma_2^{(N=1)}}{\beta_0} \right) \left( \frac{\alpha_s}{\pi} \right)^2 + O(\alpha_s^3) \quad (6)$$

where  $\beta_0$  and  $\beta_1$  are the first two scheme scheme-independent coefficients of the QCD  $\beta$ -function, namely

$$\beta_0 = \left( \frac{11}{3} C_A - \frac{2}{3} N_F \right) \quad (7)$$

$$\beta_1 = \left( \frac{34}{3} C_A^2 - 2 C_F N_F - \frac{10}{3} C_A N_F \right) \quad (8)$$

with  $N_F$  active flavours and Casimir operators  $C_F = (N_c^2 - 1)/(2N_c)$  and  $C_A = N_c$ , in the fundamental and adjoint representation of  $SU(N_c)$ . The one-loop anomalous dimension term vanishes and the leading correction to Eq.(6) comes from the scheme-independent two-loop contribution to the anomalous dimension function

$$\gamma_1^{N=1} = -4(C_F^2 - C_A C_F/2)[13 + 8\zeta(3) - 12\zeta(2)] \quad (9)$$

which was calculated in Refs.[14, 15]. Notice the appearance of the distinctive non-planar colour factor  $(C_F^2 - C_A C_F/2) = O(N_c^0)$ , which exhibits  $O(1/N_c^2)$  suppression at large- $N_c$ , in comparison with the individual weights of planar two-loop diagrams, namely  $C_F^2$  and  $C_F C_A$ , that are cancelling in the expression for  $\gamma_1^{N=1}$ .

In the  $\overline{\text{MS}}$ -like schemes the analytical expression for  $\gamma_2^{N=1}$  was obtained in [1] using a long-awaited determination of three-loop non-singlet splitting functions, made in Ref.[16], and the results of the work [17].

The result for  $\gamma_2^{N=1}$  reads [1]:

$$\begin{aligned}
\gamma_2^{N=1} &= (C_F^2 - C_A C_F/2) \left\{ C_F \left[ 290 - 248\zeta(2) + 656\zeta(3) - 1488\zeta(4) + 832\zeta(5) + 192\zeta(2)\zeta(3) \right] \right. \\
&\quad + C_A \left[ \frac{1081}{9} + \frac{980}{3}\zeta(2) - \frac{12856}{9}\zeta(3) + \frac{4232}{3}\zeta(4) - 448\zeta(5) - 192\zeta(2)\zeta(3) \right] \\
&\quad \left. + N_F \left[ -\frac{304}{9} - \frac{176}{3}\zeta(2) + \frac{1792}{9}\zeta(3) - \frac{272}{3}\zeta(4) \right] \right\} \\
&\approx 161.713785 - 2.429260 N_F \quad .
\end{aligned} \tag{10}$$

Notice the appearance in  $\gamma_2^{N=1}$  of three non-planar factors, namely  $C_F^2(C_F - C_A/2)$ ,  $C_F C_A(C_F - C_A/2)$  and  $C_F(C_F - C_A/2)N_F$ . These results of Eq. (10) are generalising the observation of non-planarity of  $\gamma_1^{N=1}$ -term of anomalous dimension function to three-loops and may be considered as the first non-obvious argument in favour of the correctness of definite results of Ref. [16].

The additional perturbative contribution to Eq. (5) comes from radiative corrections to the coefficient function

$$C^{(l)}(\alpha_s) = \frac{1}{3} \left[ 1 + C_1^{(l)N=1} \left( \frac{\alpha_s}{\pi} \right) + C_2^{(l)N=1} \left( \frac{\alpha_s}{\pi} \right)^2 + O(\alpha_s^3) \right] \tag{11}$$

where  $C_1^{(l)N=1}=0$ . The numerical expression for  $C_2^{(l)N=1}$ , namely

$$C_2^{(l)N=1} = 3.695 C_F^2 - 1.847 C_A C_F \tag{12}$$

was obtained in Ref. [18] by numerical integration of the two-loop expression for the non-singlet coefficient function of the DGLAP equation [19] calculated in the  $x$ -space in Ref. [20]. Note, that it was not realized in Ref. [18] that Eq. (12) has the same non-planar structure as in the expression (9) for  $\gamma_1^{N=1}$ . This fact was demonstrated in Ref. [1], where the following analytical result for  $C_2^{(l)N=1}$  was obtained:

$$C_2^{(l)N=1} = (C_F^2 - C_A C_F/2) \left[ -\frac{141}{32} + \frac{21}{4}\zeta(2) - \frac{45}{4}\zeta(3) + 12\zeta(4) \right] \quad . \tag{13}$$

As was noticed by G. Grunberg, the  $\overline{MS}$ -scheme result for Eq. (13) is scheme-dependent.

However, this observation does not affect the general feature of non-planarity of the  $O(\alpha_s^2)$  correction to  $I_G^v$ . Indeed, the transformation of the  $\alpha_s$ -corrections in Eq. (5) to another  $MS$ -like scheme, which has the same expression for  $\gamma_2^{N=1}$ , can be done with the help of the shift

$$\frac{\alpha_s(Q^2)}{\pi} = \frac{\alpha'_s(Q^2)}{\pi} + \beta_0 \Delta \left( \frac{\alpha'_s(Q^2)}{\pi} \right)^2 \tag{14}$$

where  $\Delta$  is the concrete  $N_c$ -independent number, which is defined by the logarithm from the ratio of regularisation scales  $\mu_{\overline{MS}}^2$  and  $\mu_{MS-like}^2$ . Thus, the general  $MS$ -like scheme expression for the coefficient  $C_2^{(l)N=1}$  takes the following form

$$C_{2 \overline{MS-like}}^{(l)N=1} = C_{2 \overline{MS}}^{(l)N=1} + \gamma_1^{N=1} \Delta \tag{15}$$

where both  $C_2^{(l)N=1}$  and  $\gamma_1^{N=1}$  have the same non-planar group weight  $C_F(C_F - C_A/2)$ . The transformation to other schemes, like MOM-schemes, are more delicate. Indeed, they affect the value of  $\gamma_2^{N=1}$  and may contain gauge-dependence. In view of this we are avoiding their consideration. However, we hope, that these transformations will not spoil the non-planar structure of the  $O(\alpha_s^2)$  approximation for  $I_G^v$ , found in Ref. [1].

Taking into account the feature, that at  $N_c \rightarrow \infty$   $\alpha_s/\pi = 4/(\beta_0 \ln(Q^2/\Lambda^2))$  and  $\beta_0 = (11/3)N_c$ , we get the following expression for  $I_G^v$  in the perturbative sector

$$I_G^v = \frac{1}{3} \left( 1 + O(1/N_c^2) \right) \quad . \quad (16)$$

In the non-perturbative sector the ratio of the twist-4 ( $1/Q^2$ )-corrections of the Gottfried and say Gross-Llewellyn Smith sum rule [21], defined as

$$I_{GLS} = \int_0^1 \frac{dx}{x} \left[ x F_3^{\nu p}(x, Q^2) + x F_3^{\bar{\nu} p}(x, Q^2) \right] \quad , \quad (17)$$

can be estimated using renormalon calculus (for a review see e.g. [22]). In the case of non-planar graphs, contributing to  $I_G$ , the typical renormalon chain insertions into the one-gluon line of the quark-gluon ladder graph should be crossed by the undressed second gluon line. As the result, it is expected in Ref. [1] that at  $N_c \rightarrow \infty$  the higher-twist contributions to the Gottfried sum rule is suppressed by a factor

$$\frac{\alpha_s}{\pi N_c} \sim \frac{1}{N_c^2 \ln(Q^2/\Lambda^2)} \quad . \quad (18)$$

relative to comparable effects in the Gross-Llewellyn Smith sum rule.

Using this estimate of Ref.[1], one may conclude, that in the limit of  $N_c \rightarrow \infty$  the Gottfried sum rule respects the isospin symmetry both in perturbative and non-perturbative sectors and is related to the Adler sum rule as

$$I_G^v = \frac{2}{3} I_A \left( 1 + O\left(\frac{1}{N_c^2}\right) \right) \quad . \quad (19)$$

However, in the real world, where  $N_c = 3$ , there are experimental indications, that in the nucleon sea there are isospin-breaking effects, which generate light-quark flavour asymmetry in the definite  $x$ -region and that  $\bar{u}(x, Q^2) < \bar{d}(x, Q^2)$  (for a review of the developing experimental situation see Ref. [10]-[12]). This, in its turn, necessitates the modification of the parton representation of the Gottfried sum rule following the definition of  $I_G$  in Eq. (4). It is interesting, that the first indications to the violation of the quark-parton model prediction  $I_G = 1/3$  and the necessity of incorporation of light-quark flavour asymmetry in partonic language came from the results of rather old SLAC experiment of Ref. [23]. However, the huge error-bars of these data and the appearance of EMC and BCDMS extractions of the Gottfried sum rule (see Refs. [24],[25]), which gave no obvious indications to the existence of light-quark flavour asymmetry, resulted in the fact that in spite of the appearance of first theoretical considerations of the possibility that  $\bar{u}(x, Q^2) \neq \bar{d}(x, Q^2)$  (see in particular the review of Ref. [10]), this non-perturbative effect was not incorporated

into early sets of parton distribution functions. At present this drawback is eliminated (see Refs. [26]- [29]). Moreover, there are some additional theoretical arguments, which indicate that at  $\bar{u}(x) < \bar{d}(x)$ . Some of them, most related to the considerations of Ref. [1], will be discussed below.

## 4. Theoretical considerations

It is interesting that perturbative QCD considerations of Ref.[14], which were based on the foundation, that the second coefficient of related anomalous dimension  $\gamma_1^{N=1}$  is non-zero, were among first theoretical arguments of the existence of light-quark flavour asymmetry in the nucleon sea. However, noticed by the authors of [14] effect non-planarity of  $\gamma_1^{N=1}$  was not related to large- $N_c$  expansion language. Unfortunately, this important work of Ref.[14], which contributed to the understanding of the necessity of introduction of light-quark flavour asymmetry in the parton distributions, was also forgotten in the definite moment (probably, the effect discussed in Ref.[14] was considered to be numerically not essential).

Other theoretical evaluations of light-quark flavour asymmetry contributions were discussed in the review reports of Ref. [10],[11]. Here we will mention the works, where the non-perturbative QCD methods were used. Among these methods is the developed in Ref. [30] instanton model and essentially based on the large- $N_c$  expansion chiral soliton model of Ref. [31], which was used in Ref. [32]. in estimates of the measure of light-quark flavour asymmetry in the nucleon sea.

Since in Ref. [1] and in some other discussions presented above large- $N_c$  expansion approach was essentially used, it is reasonably to think that the considerations of Ref. [1] and Ref. [32] may be compatible. In the latter case the values of  $I_G$  between 0.219 and 0.178 were obtained for a range of constituents quark mass between 350 and 420 MeV, in fair agreement with the announced NMC result  $I_G^{exp} = 0.235 \pm 0.026$  at  $Q^2 = 4 \text{ GeV}^2$  [33]. These values for  $I_G$  are essentially based on contribution

$$\frac{1}{2}(3I_G - 1) = \int_0^1 dx (\bar{u}(x) - \bar{d}(x)) = O(N_c^0) \quad . \quad (20)$$

estimated in Ref. [32]. It is worth to note, that for the constituent quark mass  $M=350$  MeV the  $x$ -behaviour for the difference of  $x[\bar{u}(x) - \bar{d}(x)]$  turned out to be in rather good agreement with the  $x$ -behaviours of this quantity calculated with the help of next-to-leading order (NLO) GRV parameterisation. Thus, at the NLO level one is able to describe at the qualitative level the existence of light-quark flavour asymmetry using the method of Ref. [32], essentially based on the large- $N_c$  expansion.

The comparison of the results of Ref. [1] with the ones of Ref. [32] generate several interesting to our mind questions. In conclusion let us mention several ones.

1. Does typical non-planar structure of the perturbative series for  $I_G^v$ , observed at the  $O(\alpha_s^2)$ -level, is continuing to manifest itself in higher orders ?

2. What are theoretical and possible phenomenological consequences of the regular non-planar structure of the considered perturbative series ?
3. Is there any theoretical relation between large- $N_c^2$  suppressed results of Ref. [1] and the existence of light-quark flavour asymmetry in the nucleon sea ?
4. What is the real value of the measure of light-quark flavour asymmetry, defined in Eq. (20) ?
5. Does this non-perturbative quantity is  $Q^2$ -dependent ?

Future will show, whether it will be possible to find answers on at least some of the questions given above and thus to understand the perturbative QCD puzzle, discovered in Ref. [1].

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